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Area-Integrated Heat Rates for Several Axisymmetric Vehicles

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Averaging coefficients that allow rapid estimates of area-integrated heat rates are determined for some configurations of practical interest. The values of these averaging coefficients lie in the range from 0.68 to 1.00.

Nomenclature

A = area
h = specific enthalpy
K = averaging coefficient defined by Eq. (1)
K' = averaging coefficient defined by Eq. (2)
P = pressure
q = heating rate per unit area
q_t = heating rate, integrated over surface area
r = radial distance
R = radius
s = wetted distance measured along body surface
u = velocity
γ = ratio of specific heats
θ_c = cone half-angle
θ_s = body angle, measured between local radius vector and the tangent to the surface
μ = coefficient of viscosity
ρ = density

Subscripts

δ = edge of boundary layer
0 = stagnation
∞ = freestream
B = base of body
N = nose

In the initial stages of entry vehicle preliminary design, it is very advantageous to be able to perform rapid estimates of the area-integrated heat rates. From these estimates, decisions can be made regarding vehicle configurations and the corresponding heat protection systems to be investigated.

This may be accomplished by relating the desired heat rate to stagnation point heat rate and vehicle base area through an averaging coefficient. That is,

$$q_t = KA_Bq_0 \tag{1}$$

For some vehicle shapes, it is convenient to introduce the ratio of nose radius to base radius into the equation for the desired heat rate:

$$q_t = K'(R_N/R_B)^{1/2}A_Bq_0 \tag{2}$$

If the values of K or K' are known, total heat rates are computed easily from either Eq. (1) or (2).

To determine the values of K and K', Eqs. (1) and (2) are inverted to read

$$K = q_t/A_Bq_0 \tag{3}$$

and

$$K' = \frac{q_t}{(R_N/R_B)^{1/2}\,A_Bq_0} \tag{4}$$

The equation for laminar heat rate at any point on an axisymmetric vehicle may be expressed as[1]

$$q = f(u_\infty,\rho_0,\mu_0,h_0)R_B^{-1/2}\,F(r/R_B) \tag{5}$$

where

$$F\left(\frac{r}{R_B}\right) = \frac{(r/R_B)\,[(P_\delta/P_0)^{(\gamma+1)/\gamma} - (P_\delta/P_0)^2]^{1/2}}{\left\{\int_0^{r/R_B} (r/R_B)^2[(P_\delta/P_0)^{(\gamma+1)/\gamma} - (P_\delta/P_0)^2]^{1/2}\,[d(r/R_B)/\cos\theta]\right\}^{1/2}} \tag{6}$$

At the stagnation point of an axisymmetric vehicle, Eq. (6) is reduced to

$$F_0 = 2(R_B/R_N)^{1/2}[(\gamma - 1)/\gamma]^{1/4} \tag{7}$$

The integral of Eq. (5) taken over the surface of the vehicle gives total heat rate so that, with the aid of Eq. (5), Eq. (3) becomes

$$K = \left(\frac{2}{F_0}\right)\int_0^1 \left(\frac{r}{R_B}\right) F\,\frac{d(r/R_B)}{\cos\theta} \tag{8}$$

Substitution of Eqs. (6) and (7) into Eq. (8) results in

$$K = \left(\frac{R_N}{R_B}\right)^{1/2}\left(\frac{\gamma}{\gamma-1}\right)^{1/4}\int_0^1 \frac{(r/R_B)^2\,[(P_\delta/P_0)^{(\gamma+1)/\gamma} - (P_\delta/P_0)^2]^{1/2}\,[d(r/R_B)/\cos\theta]}{\left\{\int_0^{r/R_B} (r/R_B)^2\,[(P_\delta/P_0)^{(\gamma+1)/\gamma} - (P_\delta/P_0)^2]^{1/2}\,[d(r/R_B)/\cos\theta]\right\}^{1/2}} \tag{9}$$

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This equation may be integrated once so that

$$K = 2\left(\frac{R_N}{R_B}\right)^{1/2}\left(\frac{\gamma}{\gamma-1}\right)^{1/4}\left\{\int_0^1\left(\frac{r}{R_B}\right)^2 \times \left[\left(\frac{P_\delta}{P_0}\right)^{(\gamma+1)/\gamma} - \left(\frac{P_\delta}{P_0}\right)^2\right]^{1/2}\frac{d(r/R_B)}{\cos\theta}\right\}^{1/2} \quad (10)$$

Approximating real gas behavior of air with a specific heat ratio of 1.2 and squaring gives

$$K^2 = 9.80\left(\frac{R_N}{R_B}\right)\int_0^1\left(\frac{r}{R_B}\right)^2 \times \left[\left(\frac{P_\delta}{P_0}\right)^{1.833} - \left(\frac{P_\delta}{P_0}\right)\right]^{1/2}\frac{d(r/R_B)}{\cos\theta} \quad (11)$$

Similarly, the equation for K' is

$$K'^2 = 9.80\int_0^1\left(\frac{r}{R_B}\right)^2 \times \left[\left(\frac{P_\delta}{P_0}\right)^{1.833} - \left(\frac{P_\delta}{P_0}\right)^2\right]^{1/2}\frac{d(r/R_B)}{\cos\theta} \quad (12)$$

The averaging coefficients K and K' may be obtained by numerical integration of Eqs. (11) and (12), respectively.

Spherical Segments

Equation (11) is used to compute values of the averaging coefficient for spherical segments since, for this class of shapes, the coefficient K has a smaller range of values than K'. To perform these computations, Newtonian pressure distribution is used. That is,

$$P_\delta/P_0 = \cos^2\theta \quad (13)$$

From the geometry of the configuration (see Fig. 1),

$$r/R_B = (R_N/R_B)\sin\theta \quad (14)$$

and

$$R_B/R_N = \sin\theta_s \quad (15)$$

Substitution of Eqs. (13–15) into Eq. (11) results in

$$K^2 = \frac{9.80}{\sin^4\theta_s}\int_0^{\sin\theta_s}\sin^2\theta[(\cos^2\theta)^{0.833} - \cos^2\theta]^{1/2}\, d(\sin\theta) \quad (16)$$

or

$$K^2 = (9.80/\sin^4\theta_s)G_1(\theta_s) \quad (17)$$

where

$$G_1(\theta_s) = \int_0^{\sin\theta_s}\sin^2\theta[(\cos^2\theta)^{0.833} - \cos^2\theta]^{1/2}\, d(\sin\theta) \quad (18)$$

The function G_1 is presented in Fig. 2 as a function of θ_s.

The values of the averaging coefficient K for spherical segments was computed from Eq. (17) using the values of G_1 obtained from numerical integration of Eq. (18). In Fig. 1, K is presented as a function of R_B/R_N. An analytical expression that is within $\pm$ 0.5% of the curve over the entire range of R_B/R_N except between 0.95 and 1.0, where the error is less than 3%, is

$$K = 1 - 0.128\,(R_B/R_N)^2 \quad (19)$$

If a $K = 0.95$ is used, the error will be less than 5% for R_B/R_N between zero and 0.85.

Sphere-Cones

To compute values of the averaging coefficient for tangent sphere-cone configurations, Eq. (12) is used, since for these

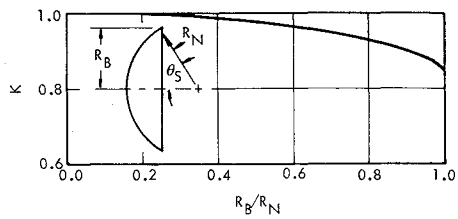

Fig. 1 Averaging coefficient for spherical segments.

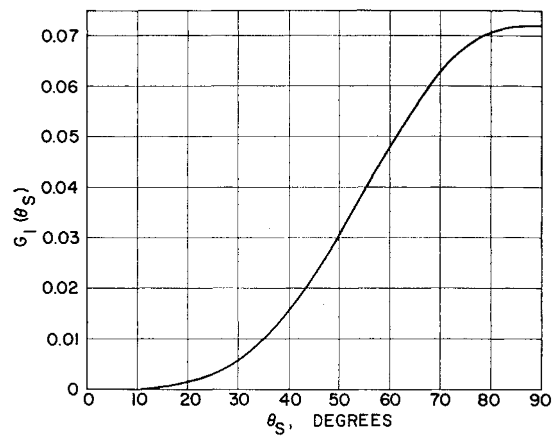

Fig. 2 G_1 as a function of θ_s.

shapes it is the coefficient K' which has the smaller range of values. It is convenient to divide the integration into two parts, one for the spherical portion of the vehicle and the other for the conical portion. If Newtonian pressure is used, the first portion of Eq. (12) is similar to the spherical segment equation, so that the equation may be written as

$$K'^2 = 9.80\left\{\left(\frac{R_N}{R_B}\right)^3 G_1(\theta_s) + \int_{(R_N/R_B)\sin\theta_s}^1\left(\frac{r}{R_B}\right)^2 \times [(\cos^2\theta_s)^{0.833} - \cos^2\theta_s]^{1/2}\, d\left(\frac{r}{R_B}\right)\right\} \quad (20)$$

By performing the indicated integration, this equation becomes

$$K'^2 = 9.80\,\{(R_N/R_B)^3 G_1(\theta_s) + \tfrac{1}{3}[1 - (R_N/R_B)^3\sin^3\theta_s] \times [(\cos^2\theta_s)^{0.833} - \cos^2\theta_s]^{1/2}\} \quad (21)$$

It is usual to describe a tangent sphere-cone configuration by its cone half-angle θ_c and its base to nose radius ratio. In terms of θ_c, Eq. (21) is written as

$$K'^2 = 9.80\,\{(R_N/R_B)^3 G_1(90° - \theta_c) + \tfrac{1}{3}[1 - (R_N/R_B)^3\cos^3\theta_c]\,[(\sin^2\theta_c)^{0.833} - \sin^2\theta_c]^{1/2}\} \quad (22)$$

Values of K' have been computed for various values of (R_N/R_B) and θ_c and are presented in Figs. 3 and 4. Figure 3 presents curves of averaging coefficient with cone half-angle as the independent variable and base to nose radius ratio as the parameter. Sphere-cone configurations with half-angles less than 10° or greater than 60° generally are considered too slender or too blunt, respectively, to be of practical interest. Thus the curves for cone half-angles between 10° and 60° have been replotted in Fig. 4 with base to nose radius ratio as the independent variable. As may be seen from this figure, for each cone half-angle an average of K' will be in error by less than 8%. Thus, K' can be written as a function of cone half-angle only. An analytical expression for this relation which increases the error by only 1% (i.e., from 8% to 9%) is

$$K' = 0.91\cos^2(\theta_c - 35°) \quad (23)$$

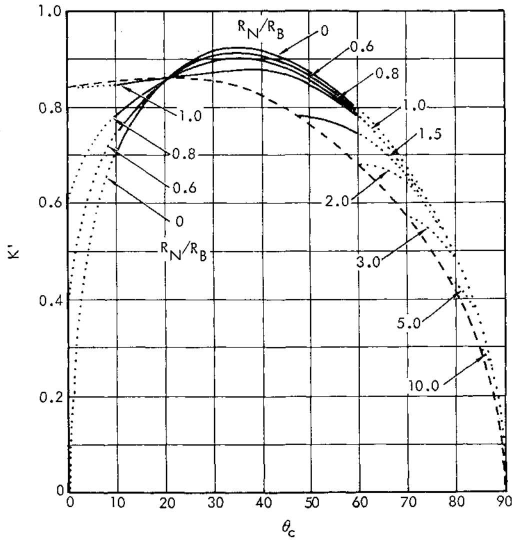

Fig. 3 Averaging coefficient for sphere-cones as a function of cone half-angle.

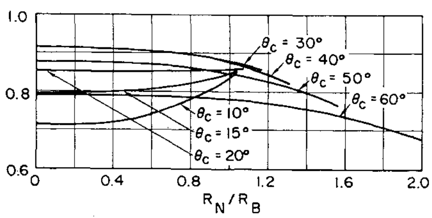

Fig. 4 Averaging coefficient for sphere-cones as a function of base to nose radius ratio.

A computation of K' has been made for a sphere-cone with a cone half-angle of 20° and R_N/R_B of $\frac{1}{3}$ using a more realistic pressure distribution obtained by the method of characteristics. The computed value of K' was 5% less than that computed using Newtonian pressure distribution. For sphere-cones with greater cone half-angles, the difference between the two pressure distributions is less, so that the error in K' as presented also is less.

Summary

The averaging coefficient K for spherical segments varies only from 1.00 to 0.84, and an error of less than 5% in the total heat rate occurs for base to nose radius ratios between zero and 0.85 if a K of 0.95 is used. Thus, for spherical segments,

$$q_t = 0.95\, A_B q_0 \qquad 0 < R_B/R_N < 0.85 \tag{24}$$

is a reasonable equation to use for preliminary design estimates. A slightly more complex equation for total heat rate which is in error by less than 3% and includes all spherical segments through hemispheres is

$$q_t = [1 - 0.128\,(R_B/R_N)^2]\, A_B q_0 \tag{25}$$

For vehicles that are sphere-cones with half-angles between 10° and 60°, the total heat rates may be computed using the equation

$$q_t = 0.91\,(R_N/R_B)^{1/2}\, A_B q_0 \cos^2(\theta_c - 35°) \tag{26}$$

Less than 9% error results from using this simple equation.

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Analysis of the Flow and Heat Transfer Processes in a Tube Arc for Heating a Gas Stream

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An approximate analysis of the flow and heat transfer processes in a tube arc is presented, the objective of the analysis being to obtain a means for computing the relations between the operating variables, the geometry, and the parameters of the flow exhausting from the device. The analysis employs the approximations usually made for free jet flows and boundary layers. It is assumed that a cool layer of gas wets the wall of the tube. The results, therefore, would be expected to apply for those ranges of the operating variables of a tube arc where the latter condition occurs.

1. Introduction

THIS note discusses methods for analyzing the flow and heat transfer processes in a tube arc, i.e., an electric arc that passes through a narrow cylindrical tube, concurrently with a flow of gas to be heated by the arc discharge. Referring to Fig. 1, the gas to be heated enters the tube at station 1. A portion of the gas is heated by the arc discharge, and the unheated balance of the gas passes near the wall of the tube. The thermal boundary, denoted by δ_t in Fig. 1, encloses the gas heated by the arc discharge.

The following principal assumptions are made in the analysis:

1) The gas is inviscid and in local thermal equilibrium.
2) The flow is steady, laminar, and axisymmetric.
3) The radial velocity component is much smaller than the axial velocity component, and there is no component of velocity in the tangential direction.
4) Ohm's law is valid.
5) Energy transport by radiation is neglected.
6) The Mach number is small.

For clarity, the basic method employed for analyzing the tube arc is first applied herein to an unconfined arc that is subjected to forced convection along its axis.

2. Analysis of an Unconfined Arc in Forced Convection

Figure 2 illustrates schematically the unconfined arc to be discussed. The velocity V of the gas far from the axis of the arc and the pressure gradient dp/dz are given functions of the axial coordinate z. The electric current I, the mass flow rate $\langle m \rangle$, the average rate of energy transport $\langle mh \rangle$, and the average rate of momentum transport $\langle mV \rangle$ through the arc are considered to be known at the axial position z_0. The parameters of the flow and the enthalpy distribution at any position along the arc are to be determined.

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